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SELF-SIMILAR MOTION OF AN IONIZED GAS EXPELLED BY A MAGNETIC PISTON

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The motion of a gas in plasma accelerators and high-current discharges, under the conditions of the skin effect, can be represented as its ejection by a magnetic piston under the action of a given current flow along the surface. Such a model was first proposed in [1] to explain the pinch effect. In the initial stage, the law of the rise in the current is approximated rather well by a linear function of the time, and the magnetic field, by a quadratic law: $p = Ct^n$, where $n = 2$; $C = \text{const}$. Under the usual conditions of an experiment, the magnetic pressure is much greater than the initial pressure of the gas, and the latter can be neglected. In this case, the motion of the gas is self-similar. An analogous problem for a given law of change in the velocity of the piston was discussed earlier [2, 3].

We shall assume the gas to be ideal and monoatomic, and the process to be adiabatic. The determining parameters in the problem will be the coordinate r, the time t, the density of the unperturbed gas ρ_1, and the constant C, determining the law of change in the pressure at the piston (the initial velocity $v_1 = 0$ and the initial pressure $p_1 = 0$). From these parameters, a single dimensionless variable can be obtained

$$\lambda = \sqrt{\frac{C}{\rho_1}}\frac{t^{n/2+1}}{r} = \sqrt{\frac{C}{\rho_1}}\frac{t^{m+1}}{r}, \quad m = \frac{n}{2}.$$

For the velocity, the density, and the pressure, we introduce the dimensionless functions V, R, P in the following manner:

$$v = \frac{r}{t}V(\lambda), \quad \rho = \rho_1 R(\lambda), \quad p = \frac{\rho_1 r^2}{t^2}P(\lambda), \quad z = \frac{\gamma P}{R},$$

then, the system of hydrodynamic equations is brought into the form [4]

$$\frac{dz}{dV} = z\frac{[2(V-1)+\nu(\gamma-1)V](V-m-1)-(\gamma-1)V(V-1)(V-m-1)-\left[2(V-1)-2m\frac{\gamma-1}{\gamma}\right]z}{(V-m-1)[V(V-1)(V-m-1)-(2m/\gamma+\nu V)z]}; \tag{1}$$

$$\frac{d\ln\lambda}{dV} = \frac{(V-m-1)^2-z}{V(V-1)(V-m-1)-(2m/\gamma+\nu V)z}; \tag{2}$$

$$\frac{d\ln R}{d\ln\lambda}(V-m-1) = \frac{V(V-1)(V-m-1)-(2m/\gamma+\nu V)z}{z-(V-m-1)^2}+\nu V, \tag{3}$$

where γ is the ratio of the specific heat capacities; $\nu = 1, 2, 3$, respectively, for plane, cylindrical, and spherical symmetry.

Let us examine the additional conditions which arise due to the presence of the surface of a strong discontinuity ahead of the piston. We note that, in the shock wave, r is a function of t. Consequently, the determining parameters in the shock wave will be t, ρ_1, and C, from which it is impossible to form a dimensionless quantity. Therefore, at the shock wave

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$$\lambda_2 = \sqrt{\frac{C}{\rho_1}} \frac{t^{m+1}}{r_2} = \text{const},$$

from which the coordinate and velocity of the shock wave

$$r_2 = \sqrt{\frac{C}{\rho_1}} \frac{t^{m+1}}{\lambda_2}, \quad D = \frac{dr_2}{dt} = (m+1)\frac{r_2}{t}. \tag{4}$$

Taking into consideration that a state of rest corresponds to the point $V_1 = 0$, $P_1 = 0$, $R_1 = 1$, $z_1 = 0$, from the conditions of the conservation of mass, momentum, and energy, with a passage through the surface of a strong discontinuity, we obtain [1]

$$V_2 = \frac{2(m+1)}{\gamma}, \quad z_2 = (m+1)^2 \frac{2\gamma(\gamma-1)}{(\gamma+1)^2}, \quad P_2 = \frac{2(m+1)^2}{\gamma+1}, \quad R_2 = \frac{\gamma+1}{\gamma-1} \tag{5}$$

(the subscript 2 denotes values behind the shock wave).

Let us examine the boundary conditions at the piston. We have

$$\begin{gathered} v^* = \frac{dr^*}{dt} = \frac{r^*}{t} V^*, \\ p^* = \rho_1 \frac{r^{*2}}{t^2} P(\lambda^*) = \rho_1 \frac{r^{*2}}{t^2} P^* = Ct^n, \end{gathered} \tag{6}$$

from which

$$r^* = \sqrt{\frac{C}{\rho_1}} \frac{t^{m+1}}{\sqrt{P^*}}.$$

Then

$$v^* = (m+1)\sqrt{\frac{C}{\rho_1}} \frac{t^m}{\sqrt{P^*}} = C_1 t^m. \tag{7}$$

We go over to the new dimensionless variable

$$\lambda = C_1 t^{m+1}/r. \tag{8}$$

The problem then reduces to an investigation of the self-similar, not fully established motion of a gas, expelled by a piston, whose velocity varies according to a power law: $v^* = C_1 t^m$, $C_1 = (m+1)\sqrt{C/\rho_1 P^*}$, $m = n/2$ (the equation naturally retains the same form). This problem was solved in [2, 3] for several values of m and ν. In distinction from these pieces of work, the constant C_1 is unknown in the present case; however, the solution of the problem is not complicated in practice, since the value of C_1 is required only for determination of the scale of the dimensional quantities, while, for the solution of the system (1)-(3), it is not essential, since it enters neither into the equations nor into the expressions for the boundary conditions.

From (6)-(8), we obtain the boundary condition at the piston

$$V^* = m+1, \lambda^* = m+1.$$

Further, let us consider the practically interesting case of plane symmetry ($\nu = 1$ and $m = 1$). The behavior of the integral curves of Eq. (1) in the plane Yz for this case is shown in Fig. 1 (0(0,0) is a node; C(2,0) is a node; D(2,00) is a saddle point; G(1,0) is a node; F($2/(\gamma+1)$, $\gamma^2(\gamma-1)/(\gamma+1)^2(\gamma+0.5)$ is a saddle point). The arrows show the direction of the rise in the parameter λ along the integral curves (at the parabola $z = (V-2)^2$, $\lambda = \lambda_{min}$). Since, with a change in r from the piston to infinity, λ decreases monotonically from $\lambda^* = 2$ to 0 (the point of rest corresponds to the point 0(0,0); a continuous passage through the parabola $z = (V-2)^2$ is physically impossible. Therefore, we can pass from a state of rest to the piston only through the shock wave. Thus, in the plane Vz we have the following picture of the motion. From infinity, the point corresponding to a state of rest 0(0,0) goes over jumpwise to a point with the coordinates (V_2, z_2) (5); then, it moves along the integral curve up to intersection with the straight line V = 2, corresponding to particles of the gas in contact with the piston. This integral curve is shown in Fig. 1 by the heavy line ($\gamma = 5/3$).

The system of equations (1)-(3) was integrated numerically. Equation (1) was integrated by the Adams method from the point (V_2, z_2) to the point (2,0). We note that, with $V \to 2$ $z = C_z(2-V)^{1/(1+\gamma)} \to 0$, the constant C_z is determined from values of z obtained by numerical integration.

Equation (2) was integrated using the Simpson formula in a reverse direction, since the value of λ is known at the piston and unknown at the shock wave. Equation (3) was brought into the form

$$\frac{d\ln R'}{dV} = \frac{\gamma}{\gamma+1}\frac{1}{2-V} + \frac{V}{V-2}\frac{(V-2)^2 - z}{(V-2)(V-1)V - (2/\gamma - V)z};$$

where $R' = R(2 - V)^{1/(1+\gamma)}$, and was integrated from the point (V_2, R_2) up to the piston, also by the Simpson formula. Replacement of variables makes it possible to achieve great exactness, since, with $V \to 2$ $R = C_R (2 - V)^{-1/(1+\gamma)} \to \infty$. The constant C_R is determined from values of R, obtained by numerical integration.

Knowing the values of z and R, values of P can be calculated for all V. At the shock wave

$$P_2 = R_2 z_2/\gamma = 8/(\gamma + 1).$$

To determine the value of P* at the piston, we use the asymptotic of z and R with $V \to 2$:

$$P^* = (zR/\gamma)_{V=V^*} = C_z C_R/\gamma.$$

As the result of a numerical solution, the values $P^* = 6.55$ and $\lambda_2 = 1.73$ were obtained.

Knowing R, z, and P for all values of V between the shock wave and the piston ($1.5 \leqslant V \leqslant 2$), we can go over from dimensionless quantities to dimensional. We shall find the ratios of the velocity v, the pressure p, the density ρ, and the temperature T to their values at the shock wave (Fig. 2):

$$\frac{v}{v_2} = \frac{\gamma+1}{4} V \frac{r}{r_2}, \quad \frac{p}{p_2} = \frac{\gamma+1}{8} P \left(\frac{r}{r_2}\right)^2, \quad \frac{\rho}{\rho_2} = \frac{\gamma-1}{\gamma+1} R,$$

$$T/T_2 = \frac{(\gamma+1)^2}{2\gamma (m+1)^2 (\gamma-1)} z \left(\frac{r}{r_2}\right)^2, \quad \frac{r}{r_2} = \frac{\lambda_2}{\lambda} = \frac{1.73}{\lambda}$$

(these relative values do not depend on the constants C and ρ_1, which determine the scale).

We note that the extreme right-hand point of the curve $v/v_2(r/r_2)$ in Fig. 2 corresponds not to the velocity of the shock wave, but to the velocity of the gas behind the shock wave, which is less than the velocity of the shock wave, since $v_2 = (r_2/t)V_2$, $D = (m+1)r_2/t$, $V_2 = 1.5 < 2$. Thus, the particles of gas located at the piston cannot overtake the shock wave.

The reversion of the temperature at the piston to zero, and of the density to infinity, is connected with the fact that the initial pressure of the gas was assumed to be negligibly small. The effects of the thermal conductivity and the viscosity, which are neglected here, are considerable only for a calculation of the front of the shock wave.

The approximation of a flat magnetic piston has been used successfully for a description of the acceleration of a plasma in a coaxial gun [5], with a small gap between electrodes, where the radial structure can be neglected. The ejection of a plasma by a piston has been described by a model of a snow plow. As is well known (see, e.g., [6]), for the validity of this model it is sufficient that all the particles of the accelerated gas be identical. As can be seen from Fig. 2, under conditions of an increase of the current in the gun, the velocity of the particles from the piston to the shock wave varies only within the limits of 20%. A comparison of the velocity of the piston, obtained from the snow-plow model and the self-similar solution (7) of the equations of hydrodynamics gives the relationship $\sqrt{P^*} : \sqrt{6} = 1.04 : 1$. In the snow-plow model, the velocity is somewhat greater. This is connected with the fact that it is the velocity of the center of gravity of a plasma bunch whose size increases.

The hydrodynamic model makes it possible to determine the profile of the bunch along the direction of the motion. In the region near the piston, however, this model gives an infinite density and a zero temperature, which limits the region of its applicability. This limitation is bound up with the fact that, at the initial moment, the pressure of the piston is equal to zero, and the postulation of the smallness of the pressure of the gas at this time is not applicable. Only with the passage of the shock wave over a distance $\Delta x = p_1/\rho_1 C_1 \lambda_2$ does the pressure of the shock wave exceed the initial pressure p_1. Using an asymptotic solution near the piston, it can be shown that the relative size of this region decreases with the time according to the law

$$\frac{\Delta r}{r_2} = \frac{1}{\lambda_2} \frac{\gamma}{\gamma+1} \left(\frac{p_1}{C} \frac{P^*}{\lambda_2 C_R}\right)^{\frac{\gamma+1}{\gamma}} t^{-2\left(1+\frac{1}{\gamma}\right)},$$

and its absolute size decreases due to compression: $\Delta r \sim t^{-2/\gamma}$.

Thus, with a finite pressure p_1, the solution for the density and the temperature is valid for the greater part of the bunch, with the exception of a small region near the piston, whose absolute and relative size decreases very rapidly with the time. Near the piston, the shock wave does not heat up the gas, and the density and temperature in this region can be evaluated from the condition of the adiabaticity of the motion

$$\frac{\rho^*}{\rho_1} = \left(\frac{p^*}{p_1}\right)^{1/\gamma}, \quad \frac{T^*}{T_1} = \left(\frac{p^*}{p_1}\right)^{\frac{1}{\gamma}-1}.$$

The width of the front of the discontinuity in the density and the pressure is determined, as is well known, by the length of the free flight path (see, e.g., [7]). For typical experimental conditions [5], it is a quantity of the order of tenths of a millimeter, negligibly small in comparison with the dimensions of the acceleration zone.

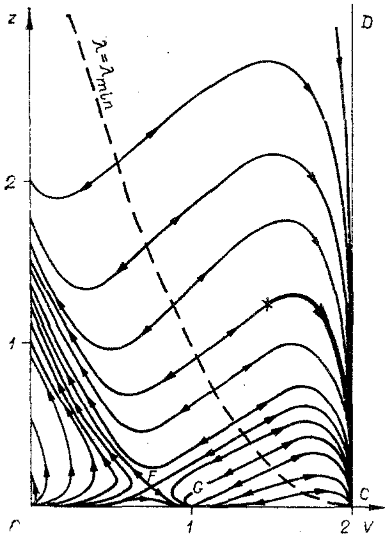

Fig. 1

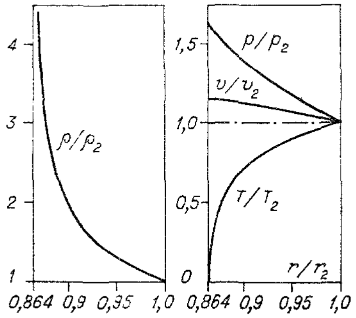

Fig. 2

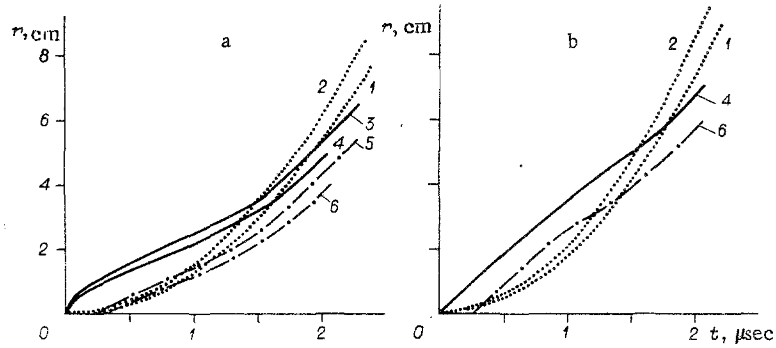

Fig. 3

As follows from (4), (7), with m = 1 the piston and the shock wave move with an equal acceleration:

$$v^* = g^* t,\ D = gt,$$

where

$$g^* = (m + 1)\sqrt{C/\rho_1 P^*};\ g = (m + 1)g^*/\lambda_2.$$

For the experimental conditions of [5], we obtain the value of the constant

$$C = 10^{-2}\frac{(dI/dt)^2}{\pi(S_2^2 - S_1^2)}\ln\frac{S_2}{S_1},$$

valid at the initial moment of a discharge in the region of a linear rise in the current, where dI/dt is the rate of rise of the current, A/sec; S_1, S_2 are the radii of the central and external coaxials. For this experiment, if, as S_1 and S_2, we take the radii of a thin filament and of the cylindrical part of the nozzle, we obtain $C = 5\cdot10^{20}$ dyn/cm^2·sec^2. In this case, the profiles of the velocity, the pressure, the density, and the temperature, given in Fig. 2, are valid with $t \gg t_1 = 60$ nsec in the whole region of a change in r/r_2, with the exception of a section near a piston of width $\Delta r/r_2 \approx 3\cdot10^{-5}/t^{2.6}$, where t is the time, μsec.

However, the piston is a hot current-carrying layer of gas, heated by Joule losses, which leads, as evaluations show, to heating of an adjacent layer of gas due to electronic thermal conductivity and the diffusion of a magnetic field with a $\Delta r/r_2 \approx 2\cdot10^{-2}/t^{2.1}$ and $5\cdot10^{-2}/t^{2.1}$, respectively. This argues that, at the start of the acceleration with $t < 0.1$ μsec, there is a possibility of the permeation of the plasma through the magnetic piston. The ratio of the characteristic times of the thermal conductivity and the skin-effect to the acceleration time are $t_T/t \approx 20t^2$ and $270t^{3.6}$, respectively, which also attests to the start of effective skimming of the plasma by the piston after 0.1 μsec.

Figure 3 shows the dependence of the path on the time for the shock wave 2 and the piston 1, as well as experimental curves of the displacement of the center of gravity of the current 4 and the front of the luminescence 3 with a pressure of 760 and 400 mm Hg (Fig. 3a and b, repsectively) [5]. It can be seen that the pressure of the current layer and the front of the luminescence, which can be connected with the magnetic piston and the shock wave, are in qualitative agreement with calculation; however, at the initial moment, the piston does not capture the gas completely and, for this reason, the experimental curves lie above the calculated. A shell is obviously formed at a distance of around 1 cm from the point of the breakdown, which is in agreement with the above-mentioned evaluations of the formation time of the piston. Therefore, lines 5 and 6 show experimental curves with a shift of the point of breakdown downward by 1 cm with respect to the point of formation of the shell. With lower pressures (30-100 mm Hg), the discharge starts in the conical part of a nozzle of variable cross section, and there is ablation of the insulator; therefore, a comparison between calculation and experiment is not justified here, since in this case the constant of the magnetic pressure depends on the time.

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DEVELOPMENT OF DYNAMIC FORMS OF BUCKLING OF ELASTOPLASTIC BEAMS WITH INTENSIVE LOADING

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1. We consider an I-beam. At the initial moment of time, an intensive longitudinal constant load is suddently applied to the beam; in the theoretical analysis, longitudinal vibrations are not taken into consideration. The intensive compressive loading is considerably greater than an Euler loading [1]. We assume that this compressive load corresponds to stresses exceeding the elastic limit. It is assumed that the bending takes place in the plane of the web, while the bending moment is taken up only by the flanges of the I-beam. A study is made of the development with time of the forms of inelastic buckling of beams with small normal bends w. Equating the sum of the internal forces with respect to the neutral line to the external moment, we find the equation of the curved axis of the I-beam [2]

$$TIw_{xxxx} + Nw_{xx} + \rho Sw_{tt} = -N(w_{0xx} + w_{1xx}). \tag{1.1}$$

Since, before loading, the freely supported beam was at rest, the initial and boundary conditions have the form

$$\begin{aligned} &w = w_t = 0,\ t = 0,\ 0 \leqslant x \leqslant L; \\ &w = w_{xx} = 0,\ x = 0,\ L,\ t \geqslant 0, \end{aligned} \tag{1.2}$$

where w is the additional normal deflection; x and t are the longitudinal coordinate and the time; N is an intensive longitudinal load; $T = 2E_1E_2/(E_1 + E_2)$ the modulus of elasticity and relief; E_2 is the reduction modulus (Fig. 1); $E = E_1$ is the tangential modulus; w_0 and w_1 are the initial regularities of the beam and the shift of the central line for cross sections of the beam; S and I are the constant area and moment of inertia of a transverse cross section; L is the length of the beam.

The function $w_1(x)$, characterizing the shift of the central line, is subject to determination. With determination of w_1, use is made of an idealized $\sigma-\epsilon$ diagram, Fig. 1, and the assumption N = const. Figure 2a shows an I-beam; the flanges of the I-beam are connected by a thin web. Three cases of the loading of beams at the initial moment of time (t = 0) are considered: 1) the beam remains elastic; 2) the stresses in the beam considerably exceed the elastic limit (point c on the idealized diagram of Fig. 1); 3) the stresses coincide with the elastic limit (point b on the idealized diagram). The

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